

Mathematics and Social Science: A Statistical Mechanics Approach to Immigration

by Pierluigi Contucci and Cristian Giardina

Is modern science able to study social matters like those related to immigration phenomena on solid mathematical grounds? Can we for instance determine cultural robustness and the causes behind abrupt changes from cultural legacies? Can we predict, cause or avoid swings? A novel approach is under investigation using the statistical mechanics formalism devised for the study of phase transitions in physics.

From the current growth rate of European immigration it is clear that in a few decades, people born outside Europe will represent a large percentage of the continent's total population. Although from an emotional point of view immigration is often perceived as a threat, to a large extent it represents an economic opportunity. When two cultures are merged, issues like the survival of each cultural identity play a major role in determining a proper and functional mutual integration. Historically there are several examples in which one cultural trait, despite being carried by only a small fraction of people, overcomes another in a relatively short time and with associated dramatic changes. In other cases, two different cultural traits may coexist peacefully for long periods of time.

Are we able to study these phenomena on solid scientific grounds? For instance, is it possible to establish what determines cultural robustness and what causes sudden changes from pre-existing cultural legacies? Can we predict or avoid such changes? From a modern epistemological perspective, can we build a 'simple' mathematical model that in terms of a few measurable parameters would provide a predictive description of the observed phenomenology at a social level?

Two research projects have taken on the challenge: a European team (CULTAPTATION) and a Strategic Team of University of Bologna. What is the idea the teams are hunting after? People interact, they exchange information, and within a given community they tend on average to imitate each other. While for a handful of people it is necessary to study all

possible decision strategies, a million subjects have a well-defined social average status that is largely independent of individual details. The science that learnt how to infer the macroscopic properties of a large number of particles from rules governing mutual interaction of small groups is called statistical mechanics; it began with the work of Boltzmann and was used to derive the laws of thermodynamics. In recent decades a statistical mechanics formalism has proven to be an excellent method for studying typical problems in which a system is described by a large number of individuals and average properties are being investigated.

With this perspective, a statistical mechanics model has been introduced that aims to describe the interaction of two groups, for instance immigrants and residents. The model assumes that the elements of the two populations of sizes N_1 and N_2 , with $N=N_1+N_2$ a very large number, interact within themselves with an interaction strength J_1 in group 1 and J_2 in group 2. Moreover a cross-group interaction with a tuneable strength J_{int} is present between individuals of different groups. The model is of mean-field type: it is assumed that individuals are nodes of a fully connected graph. By means of parameters that measure the strength of the interactions and by considering the original cultures prior to cultural meeting, it is possible to provide a quantitative description of the system. The model considered is rich of structure and able to predict, as the ratio N_1/N of the population varies, not only the coexistence of cultures but also and especially sudden changes acting with the features of phase transitions. Figure 1 displays those occurrences.

Future developments will evolve in two directions (I. Gallo, PhD thesis, in preparation). The first is to quantify the predictive value of the model by

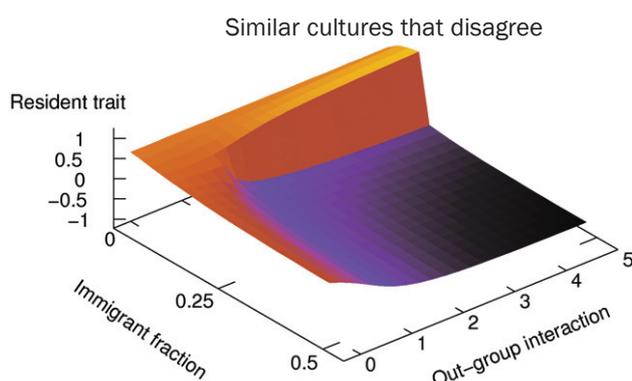


Figure 1: Influence of immigration on resident culture. The figure portrays the mean cultural trait after cultural contact as a function of the fraction of immigrants N_1/N and the strength of the cross-group interaction J_{int} . Picture source: <http://arxiv.org/abs/0712.1119>



statistical estimation of parameters starting from poll data and using the maximum likelihood methods, thereby bridging theory and experiment. The second is to extend to realistic random interaction networks the formalism used so far. There is indeed clear evidence that the social interaction network among people exhibits several topological features that appear in random networks of 'small world' and 'scale-free' types. The necessity to extend the statistical mechanics methods to complex network environments is of fundamental importance. With this aim, the conference YEP-V addressed to Young European Probabilists has been organized by the

authors. It was held in March 2008 at EURANDOM, Dutch institute member of EMS and ERCOM, to which one of the authors (C.G.) is affiliated. That initiative will continue and develop in a forthcoming conference at Banff International Research Station, Canada, in November 2009.

Links:

<http://www.intercult.su.se/cultaptation/>
<http://www.dm.unibo.it/~contucci/sf.html>
<http://arxiv.org/abs/physics/0606062>
<http://arxiv.org/abs/physics/0702076>
<http://www.eurandom.tue.nl/workshops/2008/YEPV-r/YepVmain.htm>
http://www.birs.ca/birspages.php?task=displayevent&event_id=09w5055

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The Future of Mathematics Education in Europe

by Olga Caprotti and Mika Seppälä

The level of education of their workforces determines the success of nations in global competition. Quantitative reasoning and the ability to apply mathematical methods in general will be the most important components in the skill set of tomorrow's workforce, meaning mathematics education has great strategic importance. The question of how to educate more people in mathematics, preferably with fewer resources, is an equation that cannot be solved by mathematics alone: computer science and linguistics are also needed. The WebALT eContent project has developed solutions that automate parts of mathematics instruction. Automation is the only way to improve the delivery of education, and to offer the opportunity to learn to everybody.

Learning to master certain routine tasks, such as computing with fractions, solving equations and computing limits, derivatives and integrals, forms a large part of mathematics education in secondary and early tertiary education. Training these skills produces the computational fluency and execution of procedures required, together with conceptual understanding, to support efficient problem solving. Drills and formative assessments can be delivered especially well by automatic learning systems. Computer-assisted assessment is based on advanced algorithmic exercises that are newly generated each time they are invoked. This is the most valuable aspect of e-learning materials. The WebALT project 2005-2006 (EDC-22253) developed a grammar that is able to encode these algorithmic problems so that they can be generated automatically in several European languages. This is made possible by employing Web standards (to represent the algorithmic exercises) in combination with advanced computational linguistic tools (to produce the various verbalizations). This

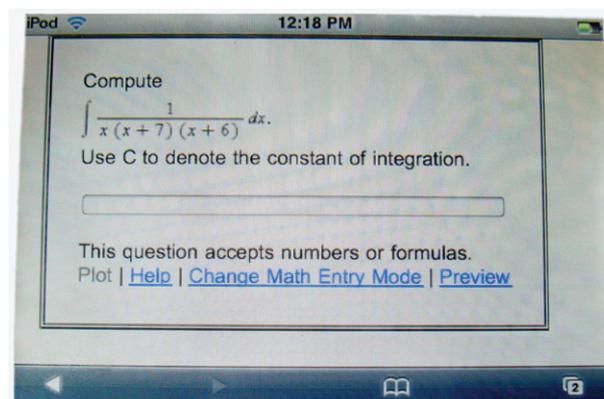


Figure 1: Maths lessons on an iPod.

technology contributes both to the preservation of the linguistic diversity and richness in Europe and to the creation of a pool of standardized tests aligned with the Bologna process. WebALT multilingual exercises are language-independent and can be adopted across borders. This multiplies the value of the content many times over.

The algorithmic problems together with high-quality supporting materials empower instructors to teach large

numbers of students with the same effort needed to teach just one small group. Grading of homework, quizzes and examinations becomes automatic, available to students any time, anywhere. Even lectures can be delivered automatically as podcasts, turning mobile devices to portable lecture halls. The WebALT eContent was coordinated by the University of Helsinki. The Technical University of Catalonia (UPC), the Technical University of Eindhoven and the University of